

# A Statistical Study of the **Compressible Energy Cascade Rate** in Solar Wind Turbulence: Parker Solar Probe Observations

M. Brodiano,<sup>1,2</sup> N. Andrés,<sup>2,3</sup> and P. Dmitruk<sup>1,2</sup>

<sup>1</sup>*Instituto de Física de Buenos Aires, CONICET-UBA, Ciudad Universitaria, 1428 Buenos Aires, Argentina*

<sup>2</sup>*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, 1428, Buenos Aires, Argentina.*

<sup>3</sup>*Instituto de Astronomía y Física del Espacio, CONICET-UBA, Ciudad Universitaria, 1428, Buenos Aires, Argentina*

(Dated: July 15, 2022)

We investigated the incompressible and compressible magnetohydrodynamic (MHD) energy cascade rates in the solar wind at different heliocentric distances. We used in situ magnetic field and plasma observations provided by the Parker Solar Probe (PSP) mission and exact relations in fully developed turbulence. **To estimate the compressible cascade rate**, we applied two recent exact relations for compressible isothermal and polytropic MHD turbulence, respectively. **Our observational results show a clear increase of the compressible and incompressible cascade rates as we get closer to the Sun.** Moreover, we obtained an increase in both isothermal and polytropic cascade rates with respect to the incompressible case as compressibility increases in the plasma. Further discussion about the relation between the compressibility and the heliocentric distance is carried out. Finally, we compared both exact relations as compressibility increases in the solar wind and although we note a slightly trend to observe larger cascades using a polytropic closure, we obtained essentially the same cascade rate in the range of compressibility observed.

## I. INTRODUCTION

The solar wind is a well-studied supersonic plasma flow, from the closest point to the Sun to the edge of the heliosphere, characterized by a turbulent energy cascade rate (Tu and Marsch, 1995; Fraternali *et al.*, 2019). The availability of in situ measurements from various orbiting spacecraft through different heliocentric distances allow a deep understanding of the essential role of turbulence from the large scales up to the kinetic scales in the solar wind plasma (Alexandrova *et al.*, 2013; Bruno and Carbone, 2013; Kiyani, Osman, and Chapman, 2015; Chen, 2016). In this sense, the Parker Solar Probe (Fox *et al.*, 2016) (PSP) mission has been exploring the inner heliosphere since 2018, approaching to the Sun with each orbit and enabling us to study the evolution of turbulence and to compare observations with theoretical predictions.

A prevailing challenge in the solar wind community is to enhance the models of the turbulence heating of the plasma, with particular focus in the near-Earth space and the magnetosphere environments (Sahraoui, Hadid, and Huang, 2020; Chen *et al.*, 2020; Huang *et al.*, 2020; Andrés *et al.*, 2019, 2021; Andrés *et al.*, 2022; Huang and Sahraoui, 2019; Hadid *et al.*, 2018; Kiyani, Osman, and Chapman, 2015; Alexandrova *et al.*, 2013; Howes *et al.*, 2012; Osman *et al.*, 2011). Observations have shown that the solar wind proton temperature tends to decrease slowly as a function of the radial distance from the Sun in contrast to the prediction of the adiabatic expansion model of the solar wind (Marsch *et al.*, 1982; Vasquez *et al.*, 2007; Pine *et al.*, 2020). While several scenarios have been proposed to explain these observations, one of the main candidate is certainly the local heating of the solar wind plasma via the turbulent cascade (Bruno and Carbone, 2013; Matthaeus and Velli, 2011). In this picture, the energy injected at the largest scales in the solar wind will cascade within the inertial range until it reaches the ion scales where it is eventually transformed into thermal or kinetic energy of the plasma particles (see Sahraoui, Hadid, and Huang (2020); Kiyani, Osman, and Chapman (2015)).

Several efforts have been made in order to provide an estimation of the energy cascade rate in the solar wind at different scales and heliocentric distances. The first exact relation use the von-Kármán-Howarth dynamical equation (von Kármán and Howarth, 1938) and it is considered one of the very few exact results in hydrodynamic (HD) turbulence theory (Frisch, 1995). This exact relation gives an expression for the energy dissipation or cascade rate  $\varepsilon$  as a function of structure functions of the turbulent fields (e.g., Monin and Yaglom, 1975; Frisch, 1995). Galtier and Banerjee (2011) and Banerjee and Galtier (2014) generalized this exact relation to compressible

isothermal and polytropic HD turbulence, respectively, under the assumption of homogeneity and in the asymptotic limit of a high Reynolds number. The authors have found the presence of a new type of term which acts on the inertial range similarly as a source or a sink for the mean energy transfer rate.

In the case of magnetized plasmas, the first proper exact relation was derived by [Politano and Pouquet \(1998a,b\)](#) for homogeneous, isotropic and incompressible magnetohydrodynamics (IMHD) turbulence. The authors recovered a scaling law for mixed third-order longitudinal structure and correlation functions. This exact law has been the subject of several numerical tests [Mininni and Pouquet \(e.g., 2009\)](#); [Boldyrev, Mason, and Cattaneo \(e.g., 2009\)](#); [Wan \*et al.\* \(e.g., 2010\)](#); it has been used for the estimation of the incompressible cascade rate in space plasmas ([Sorriso-Valvo \*et al.\*, 2007](#); [Sahraoui, 2008](#); [Coburn \*et al.\*, 2015](#); [Simakov and Chacón, 2008](#); [Masters \*et al.\*, 2008](#)) and the magnetic and kinetic Reynolds numbers ([Weygand \*et al.\*, 2007](#)) in solar wind turbulence, and for the large-scale modeling of the solar wind ([Matthaeus \*et al.\*, 1999](#); [MacBride, Smith, and Forman, 2008](#)). [Banerjee and Galtier \(2013\)](#) derived an exact relation for some two-point correlation functions of the fields for isothermal compressible magnetohydrodynamic (CMHD) turbulence, expressed in terms of flux or source terms. On the other hand, [Andrés and Sahraoui \(2017\)](#) revisited the latter work and expressed the exact law as a function of the proper plasma variables, i.e., the plasma mass density, the plasma velocity field and the compressible Alfvén velocity. This theoretical work showed that there are four different types of terms that are involved in the non-linear cascade of energy in the inertial range: the hybrid terms (which can be written either as flux or source terms), the  $\beta$ -dependent terms and the well-known flux and source terms (see, [Andrés and Banerjee \(2019\)](#); [Ferrand \*et al.\* \(2020\)](#)). Recently, [Simon and Sahraoui \(2021\)](#) proposed a more general method that allows to derive the exact relation for any turbulent isentropic flow (i.e., constant entropy). The authors demonstrated that the well-studied MHD exact laws (incompressible and isothermal) and the new (polytropic) one can be obtained as specific cases of the general exact relation when the corresponding closure is specified. These formulations for IMHD and CMHD turbulence are used in the present study to estimate the non-linear transfer of energy in the solar wind.

A first attempt to include the compressibility in estimating the energy cascade rate using in situ observations was reported by [Carbone \*et al.\* \(2009\)](#) using a phenomenological model and Ulysses observations. The authors found a significant increase in the turbulent cascade rate and a better scaling with respect to the incompressible exact law. However, those results were based on a

heuristic model, using pseudo-energies, which are not conserved in CMHD theory (Marsch and Mangeney, 1987). In the same framework, Marino *et al.* (2008) showed that the compressible turbulent cascade seems to be able to supply the energy needed to account for the local heating of the non-adiabatic solar wind. Banerjee *et al.* (2016) and Hadid, Sahraoui, and Galtier (2017) have studied the impact of the compressible fluctuations in the energy cascade in the solar wind using a reduced form of the exact relation for CMHD turbulence and in situ observations from the Time History of Events and Macroscale Interactions during Substorms (THEMIS) (Auster *et al.*, 2009) spacecraft. The authors found that the compressible fluctuations are shown to amplify by several order of magnitude the turbulent cascade rate with respect to the incompressible model. Recently, Bandyopadhyay *et al.* (2020) have computed the incompressible energy transfer rate between 35 and 55 solar radius using PSP observations during the first encounter. Andrés *et al.* (2021) extended previous observational studies computing the compressible energy transfer rate from  $\sim 0.2$  au up to  $\sim 1.7$  au, using PSP, THEMIS and Mars Atmosphere and Volatile Evolution (MAVEN) observations. The authors showed that, depending on the level of compressibility in the plasma, the different terms in the compressible exact relation were shown to have different impact in the total cascade rate (where the incompressible terms are included). Moreover, using more than 2 years of PSP observations, Andrés *et al.* (2022) studied the incompressible energy cascade rate using isotropic and anisotropic exact relations. The authors found a connection between the heliocentric distance, the local temperature of the plasma and the energy cascade rate, with a clear dominance of the perpendicular cascades over the parallel cascades as PSP approaches the Sun.

In the present paper, we use the magnetic field and plasma moments PSP observations at different heliocentric distances to compute the compressible and incompressible energy cascade rate. Our goal is to discuss the role of compressibility and heliocentric distance in the energy cascade in the solar wind. The paper is organized as follows: in section II we recall the theoretical CMHD set of equations and present briefly the main steps to derive the exact law for fully developed CMHD turbulence. In section III, we describe the data set composed of more than 3000 PSP events and the selection criteria used in the present study. In section IV, we present the main observational results. Finally, in section V we provide a summary and discussion of our main findings.

## II. THEORETICAL MODELS AND EXACT RELATIONS

### A. Compressible MHD model

The three dimensional (3D) CMHD model is given by the mass continuity equation, the momentum equation for the velocity field in which the Lorentz force is included, the induction equation for the magnetic field  $\mathbf{B}$  and the differential Gauss's law (solenoidal condition  $\nabla \cdot \mathbf{B} = 0$ ). These equations can be written as (Marsch and Mangeney, 1987),

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}), \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u}_A \cdot \nabla \mathbf{u}_A - \frac{\nabla(P + P_M)}{\rho} - \mathbf{u}_A(\nabla \cdot \mathbf{u}_A) + \mathbf{f}_k + \mathbf{d}_k, \quad (2)$$

$$\frac{\partial \mathbf{u}_A}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u}_A + \mathbf{u}_A \cdot \nabla \mathbf{u} - \frac{\mathbf{u}_A}{2}(\nabla \cdot \mathbf{u}) + \mathbf{f}_m + \mathbf{d}_m, \quad (3)$$

$$0 = \mathbf{u}_A \cdot \nabla \rho + 2\rho(\nabla \cdot \mathbf{u}_A), \quad (4)$$

where  $\mathbf{u}$  is the velocity field (assuming a zeroth background flow speed) and  $\mathbf{u}_A = \mathbf{B}/\sqrt{4\pi\rho}$  is the compressible Alfvén velocity with  $\mathbf{B}$  the total magnetic field and  $\rho$  the mass density. Note that, the time dependence enters through  $\mathbf{u}$ ,  $\mathbf{B}$  and  $\rho$ . In addition,  $P$  is the scalar isotropic pressure and  $P_M = \rho u_A^2/2$  is the magnetic pressure. In the present work, we use two different equations of state which allow us to close the hierarchy of the fluid equation (so there is no need of an energy equation): the isothermal case, i.e.,  $P = C_s^2 \rho$ , where  $C_s$  is constant sound speed and the polytropic case, i.e.,  $P = C_s \rho^\gamma$ ,  $\gamma = 5/3$  is the polytropic index and  $C_s$  is the variable sound speed. Finally,  $\mathbf{f}_{k,m}$  are the respectively mechanical and the curl of the electromotive large-scale forcing, and  $\mathbf{d}_{k,m}$  are respectively the small-scale kinetic and magnetic dissipation terms.

### B. Exact relation in CMHD turbulence

In the CMHD model, the density total energy  $E(\mathbf{x})$  and the density-weighted cross helicity  $H(\mathbf{x})$  are given by,

$$E(\mathbf{x}) = \frac{\rho}{2}(\mathbf{u} \cdot \mathbf{u} + \mathbf{u}_A \cdot \mathbf{u}_A) + \rho e, \quad (5)$$

$$H(\mathbf{x}) = \rho (\mathbf{u} \cdot \mathbf{u}_A), \quad (6)$$

where  $e$  is the internal energy. While the total energy is one of the ideal invariants, the density-weighted cross helicity is not. Nevertheless, both quantities are essential for the derivation of the

exact law in CMHD turbulence (see, [Andrés and Sahraoui, 2017](#); [Simon and Sahraoui, 2021](#)). In particular, we can define the two-point correlation function associated with the total energy, the helicity and the magnetic field by,

$$R_E(\mathbf{x}, \mathbf{x}') = \frac{\rho}{2}(\mathbf{u} \cdot \mathbf{u}' + \mathbf{u}_A \cdot \mathbf{u}'_A) + \rho e', \quad (7)$$

$$R_H(\mathbf{x}, \mathbf{x}') = \frac{\rho}{2}(\mathbf{u} \cdot \mathbf{u}'_A + \mathbf{u}_A \cdot \mathbf{u}'), \quad (8)$$

$$R_B(\mathbf{x}, \mathbf{x}') = \frac{\rho}{2}(\mathbf{u}_A \cdot \mathbf{u}'_A) \quad (9)$$

where the prime denotes field evaluation at  $\mathbf{x}' = \mathbf{x} + \boldsymbol{\ell}$  ( $\boldsymbol{\ell}$  being the displacement vector) and the angular bracket  $\langle \cdot \rangle$  denotes an ensemble average. The properties of spatial homogeneity implies that the results of averaging over a large number of realizations can be obtained equally well by averaging over a large region of space for one realization. In particular, under homogeneity assumption, the correlation functions will depend only on the vector displacement  $\boldsymbol{\ell}$ .

Using Eq. (1)-(4), the expressions (7)-(9) and following the usual assumptions for fully developed homogeneous turbulence (i.e., infinite kinetic and magnetic Reynolds numbers and a steady state with a balance between forcing and dissipation) (see, [Banerjee and Kritsuk, 2018](#); [Banerjee and Andrés, 2020](#)), an exact relation for compressible MHD turbulence can be obtained in the inertial range as,

$$-2\varepsilon_C = \frac{1}{2}\nabla_{\boldsymbol{\ell}} \cdot \mathbf{F}_C + S_C + S_H + M_{\beta}, \quad (10)$$

where  $\varepsilon_C$  is the total compressible energy cascade rate,  $\mathbf{F}_C$  is the total compressible flux (defined below) and  $S_C$ ,  $S_H$  and  $M_{\beta}$  are the so-called source, hybrid and  $\beta$ -dependent terms, respectively. For a detailed derivation and the explicit expressions of the total compressible energy cascade rate, see [Andrés and Sahraoui \(2017\)](#); [Simon and Sahraoui \(2021\)](#). It is worth mentioning that, the derivation of the exact law (10) does not require the assumption of isotropy ([Andrés et al., 2022](#)) and that it is independent of the dissipation mechanisms acting in the plasma (assuming that the dissipation acts only at the smallest scales in the system). In the present paper, in order to estimate the compressible energy cascade rate, we shall consider only the flux terms. The main reason is that the source, hybrid and  $\beta$ -dependent terms require computing the divergence of the plasma and the compressible Alfvén speeds, which can be done only using multi-spacecraft observations. Moreover, numerical results for supersonic and subsonic HD and MHD turbulence have shown that the source, hybrid and  $\beta$ -dependent terms are negligible with respect to the flux term in the inertial range (e.g., [Kritsuk, Wagner, and Norman, 2013](#); [Andrés, Galtier, and Sahraoui, 2018](#); [Andrés et al., 2019](#); [Ferrand et al., 2020](#)).

The total compressible flux is a combination of two terms of different nature, a Yaglom-like term,

$$\mathbf{F}_{1C} = \langle [\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u} + \delta(\rho\mathbf{u}_A) \cdot \delta\mathbf{u}_A] \delta\mathbf{u} - [\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u}_A + \delta\mathbf{u} \cdot \delta(\rho\mathbf{u}_A)] \delta\mathbf{u}_A \rangle, \quad (11)$$

which is the compressible generalization of the incompressible term (Politano and Pouquet, 1998b,a), and a new purely compressible flux term,

$$\mathbf{F}_{2C} = 2\langle \delta\rho\delta e\delta\mathbf{u} \rangle, \quad (12)$$

which is a new contribution to the energy cascade rate due to the presence of density fluctuations in the plasma (Andrés and Sahraoui, 2017; Simon and Sahraoui, 2021). Here, we have introduced the usual increments, i.e.,  $\delta\alpha = \alpha' - \alpha$ . It is worth mentioning that the difference between the two compressible models is in the form of the specific internal energy  $e$  given by  $e = C_{s0}^2 \ln(\rho/\rho_0)$  in the isothermal case and  $e = (C_s^2 - C_{s0}^2)/(\gamma(\gamma - 1))$  in the polytropic one. The sound speed  $C_s$  (and its mean value  $C_{s0}^2$ ) can be obtained from the perfect gas equation,  $C_s^2 = \gamma k_B T_p / m_p$ , where  $T_p$  and  $m_p$  are the proton temperature and mass, respectively, with  $\gamma = 1$  and  $\gamma = 5/3$  for the isothermal and polytropic case, respectively. Therefore, when we estimate the polytropic and isothermal cascade rate differences would arise only from the compressible flux term (12). Finally, assuming statistical isotropy, we can integrate the flux terms in (10) over a sphere of radius  $\ell$  to obtain a scalar relation for isotropic turbulence. In compact form, (10) can be written as,

$$-\frac{4}{3}\varepsilon_C\ell = F_{1C} + F_{2C}, \quad (13)$$

where  $F_{1C} + F_{2C} = (\mathbf{F}_{1C} + \mathbf{F}_{2C}) \cdot \hat{\mathbf{u}}_{sw}$  is the flux term projected into the mean plasma flow velocity field  $\mathbf{u}_{sw}$ . Note that, we called  $\varepsilon_{1C} + \varepsilon_{2C} = (-3/4\ell)(F_{1C} + F_{2C})$ .

In the incompressible limite  $\rho \rightarrow \rho_0$ , we recover the Politano and Pouquet law for fully developed incompressible MHD turbulence,

$$-\frac{4}{3}\varepsilon_I\ell = F_I, \quad (14)$$

where  $F_I$  is the projection of  $\mathbf{F}_I = \rho_0\langle[(\delta\mathbf{u}^2) + (\delta\mathbf{u}_A^2)]\delta\mathbf{u} - 2(\delta\mathbf{u} \cdot \delta\mathbf{u}_A)\delta\mathbf{u}_A\rangle$  along the mean plasma flow velocity field (Politano and Pouquet, 1998a,b). Assuming the Taylor hypothesis, i.e.,  $\ell \approx u_0\tau$ , where  $\tau$  is the time lag and  $u_0$  is obtained by averaging over each time interval, all the quantities that are indexed by 0, Eqs. (13) and (14) can be expressed as a function of the time lags  $\tau$ .

### III. PSP OBSERVATIONS AND DATA SELECTION CRITERIA

In order to estimate the compressible energy cascade rate, we employed magnetic field measurements by the FIELDS flux gate magnetometer (MAG), along with proton density, velocity and temperature data from the Solar Probe Cup (SPC) of the SWEAP instrument suite (Bale *et al.*, 2016; Fox *et al.*, 2016; Kasper *et al.*, 2016; Bale *et al.*, 2019; Kasper *et al.*, 2019; Case *et al.*, 2020). Our analysis on the PSP observations involved a time interval between November 2, 2018, and December 30, 2020. This data set was divided into a series of samples of equal duration of 60 minutes. This particular time duration ensures having at least one correlation time of the turbulent fluctuations for each particular heliocentric distance (Parashar *et al.*, 2020; Hadid, Sahraoui, and Galtier, 2017). To generate uniform time series, we re-sampled all the variables to 30 s time resolution. Thus, there are 10944 events using this data set.

In our data set, spurious spikes in the SPC moments, which are remnants of poor quality of fits, were removed. To accomplish this, consecutive filters were applied, as Figure 1 shows. First, for each event we applied a *sharp* mean value filter, which detect each spurious data and replace it with the average between the previous and the next valid observation. When we say valid observation, we are referring to a real value of the data set. Although this filter removes most of the spurious data, in some cases leaves a fictitious large data value. Therefore, we applied the well-known Hampel filter in order to detect this fictitious outliers (Davies and Gather, 1993; Liu, Shah, and Jiang, 2004; Pearson *et al.*, 2016). In few words, the Hampel filter uses a moving window implementation of predetermined size to compute the local median  $m_i$  and the local standard deviation  $s_i$ . Then, for each point  $x_i$ ,

$$y_i = \begin{cases} x_i & \text{if } |x_i - m_i| \leq n_\sigma S_i, \\ m_i & \text{if } |x_i - m_i| > n_\sigma S_i, \end{cases} \quad (15)$$

if the absolute difference of the value of that point and the local median,  $|x_i - m_i|$  is above a threshold defined as  $n_\sigma$  times the local standard deviation, the value is replaced by the median. If not, the algorithm leaves the current point unchanged and proceeds to the next point. We applied this technique using a local window of 8 points and choosing a threshold value of  $n_\sigma = 3$  (Bandyopadhyay *et al.*, 2018). This choice of parameters help eliminate most of the undetected spikes and smooth the data set probably altered by the previous filter.

In addition, we considered only intervals that did not show large fluctuations of the energy cascade rate over the large MHD scales, typically we retained events with  $std(\varepsilon_l)/mean(|\varepsilon_l|) <$

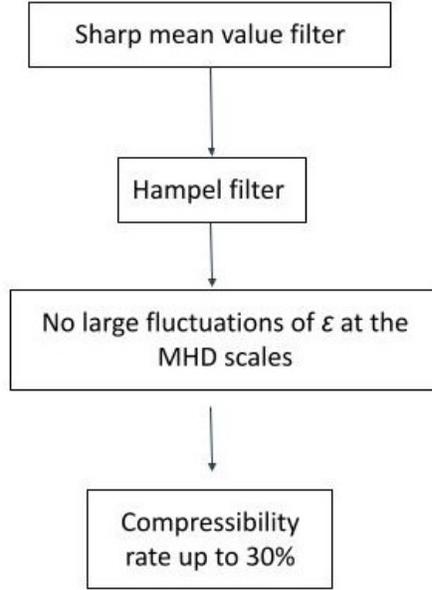


Figure 1. Schematic diagram of the filters used in order to remove spurious spikes from the data set.

0.85. Finally, we analyzed the distribution of density fluctuations of the filtered data set. Figure 2 shows the occurrence rate for all the analyzed events for the numerical density, velocity, and Alfvén velocity field absolute values and their fluctuations, respectively. We can relate the distribution of density fluctuations with the compressibility ratio defined as  $\sqrt{\langle \rho^2 \rangle - \langle \rho \rangle^2} / \langle \rho \rangle$ . Since most of the events did not show high levels of compressibility rate, we only kept the events whose compressibility reached up to 30%. This leaves us a data set of 3298 events.

#### IV. OBSERVATIONAL RESULTS

In order to compute the compressible and incompressible energy cascade rate, we constructed correlation functions of the different turbulent fields at different time lags  $\tau$  in the interval  $\tau = [30, 3480]$ s. Therefore, once we had the energy cascade as a function of the time increments and in order to quantify statistically these increments, we took the average of the absolute values in the largest MHD scales, i.e.,  $\tau = [1000, 3000]$ s to obtain representative values of each event in the large scales (Hadid, Sahraoui, and Galtier, 2017; Andrés *et al.*, 2020).

## A. The compressible and incompressible energy cascade rates

Figure 3 and 4 show the absolute value of the total compressible energy cascade rate  $\langle |\varepsilon_T| \rangle$  as a function of the absolute value of the incompressible cascade rate  $\langle |\varepsilon_I| \rangle$  for (a) the isothermal (superscript *AS*) and (b) the polytropic (superscript *SS*) model, respectively. The colorbars represent the heliocentric distance per event and the compressibility rate  $\sqrt{\langle \rho^2 \rangle - \langle \rho \rangle^2} / \langle \rho \rangle$  (in percent) of each event. We found a strong correlation between the cascade rate amplitude and the distance to the Sun. In particular, the closer to the Sun PSP is, the larger the energy cascade amplitude is. This is in agreement with previous work in the solar wind studying the incompressible energy cascade rate (e.g., [Stawarz et al., 2009, 2010](#); [Coburn et al., 2015](#); [Chen et al., 2020](#); [Andrés et al., 2022](#)). Moreover, we observed an increase in the energy cascade rate as the level of compressibility grows, reaching up to 25%. Also, we note a clear correlation between the compressible and the incompressible cascade rates. It is worth mentioning that, for low values of the cascade, there is a slight increase of the compressible cascade with respect to the incompressible one. It is worth noting that, as we are studying the total expression (13), we are not able to differentiate between the first and the second term which are the Yaglom-like term and the purely compressible one, respectively.

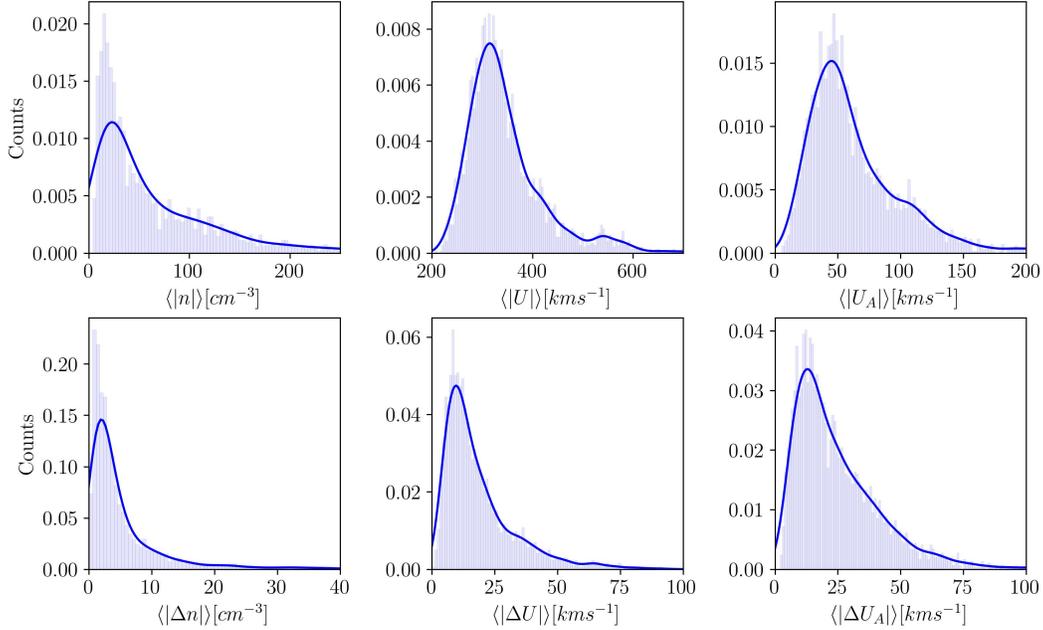


Figure 2. The occurrence rate for the proton density, the proton and Alfvén velocity absolute mean values, in the first row and the corresponding fluctuations in the second row, respectively

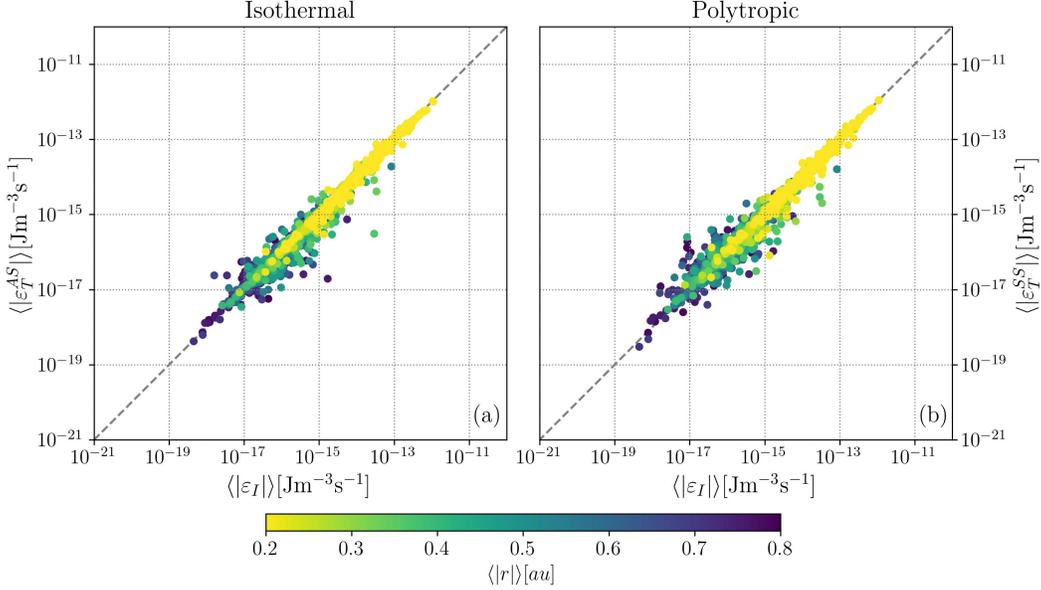


Figure 3. Total compressible energy cascade rate (absolute values) in the MHD scales as a function of the incompressible ones in the case of using an isothermal (AS) and polytropic (SS) model, respectively. The colorbar corresponds to the heliocentric distance per event.

## B. The isothermal and polytropic energy cascade components

Figure 5 and 6 show the absolute value of (a) the isothermal component  $\langle |\varepsilon_{2C}^{AS}| \rangle$  and (b) the polytropic component  $\langle |\varepsilon_{2C}^{SS}| \rangle$  as a function of the Yaglom-like compressible component  $\langle |\varepsilon_{1C}| \rangle$ , respectively. Again, the colorbar corresponds to the heliocentric distance and the compressibility percent, respectively. Although, in general, the dominant term in the MHD scales is  $\langle |\varepsilon_{1C}| \rangle$ , we noticed that even for the nearly incompressible solar wind (Compressibility  $\leq 10\%$ ), for some events the purely compressible term is relevant for a proper estimation of the total cascade. We also observed that there are some events where the energy of the compressible term exceeds the Yaglom-like term. Moreover, the level of compressibility increases up to 20-25% in those events. Therefore, when we compared the compressible and the incompressible energy cascade in Figure 4, there is an important compressible component even though most of the events tend to follow an incompressible status.

We found the same behavior of the energy cascade with the heliocentric distance as seen in Figure 3. Thus, we observe the fact that the energy cascade rates, both compressible and incompressible ones, are ordered by the heliocentric distance. Regarding the polytropic and the

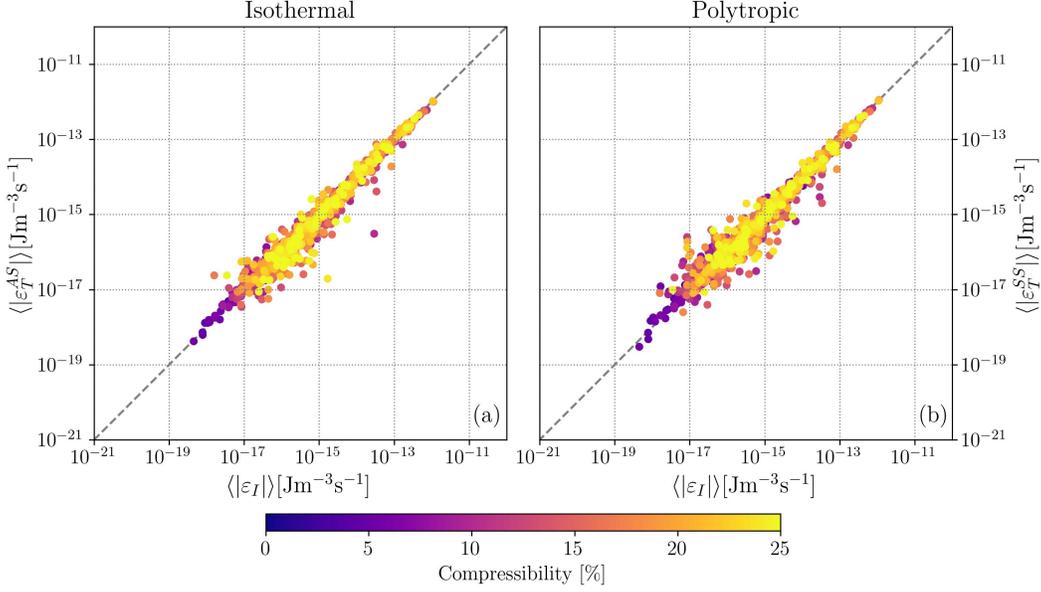


Figure 4. Total compressible energy cascade rate (absolute values) in the MHD scales as a function of the incompressible ones in the case of using an isothermal (AS) and polytropic (SS) model, respectively. The colorbar corresponds to the level of compressibility (i.e.,  $\sqrt{\langle \rho^2 \rangle - \langle \rho \rangle^2} / \langle \rho \rangle$ ) per event.

isothermal models, we noted that there is also a slight increase of the compressible term in the first case with respect to the second one. In addition, we note that there are large cascade values when we use the polytropic model.

### C. The effect of compressibility and heliocentric distance over the energy cascade rates

Figure 7 shows the bin-average of the absolute value of the energy cascade rate as a function of (a) the compressibility and (b) the heliocentric distance. It is worth mentioning that we group events according to these magnitudes and then, we segment and sort data values into bins. Finally, we took the average of each bin. In Fig. 7 (a) the colorbar corresponds to  $\langle |r| \rangle$  and in Fig. 7 (b) the colorbar corresponds to the compressibility. In particular, we compared the behavior of the total expression of the compressible cascade, the compressible components, and the incompressible cascade. We reported the standard error as the error bars. Figure 7 summarizes the results seen in previous Figs. 3-6.

As we discussed previously, we notice that as we approach to the Sun, the absolute value of the compressible energy cascade rate increases. Also, as the level of compressibility increases in the

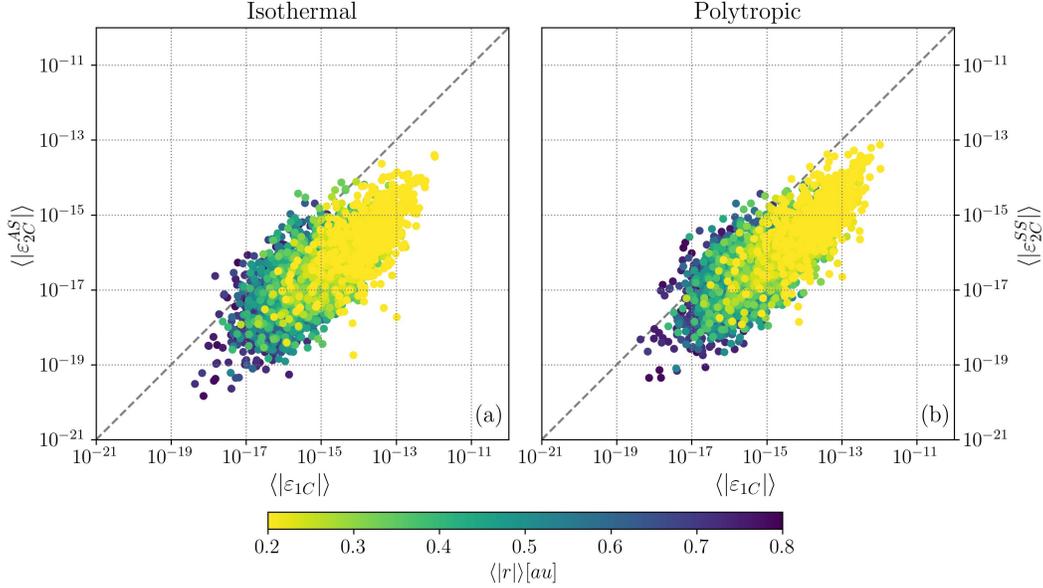


Figure 5. The compressible cascade rate component  $\langle |\varepsilon_{2C}| \rangle$  as a function of the Yaglom-like term  $\langle |\varepsilon_{1C}| \rangle$  in the MHD scales in the case of using an isothermal (AS) and polytropic (SS) model, respectively. The colorbar corresponds to the Heliocentric distance per event.

solar wind, there is an important increment of the compressible energy cascade, approximately one order of magnitude. In the case of the incompressible cascade rate, the increment of the rate is less relevant compared to the compressible one. It is worth mentioning that, there is a clear relation between the compressibility and the distance to the Sun: as we travel away from the Sun, the larger the level of compressibility is. Interestingly, here we show that, on average,  $\langle |\varepsilon_{2C}^{\text{SS}}| \rangle > \langle |\varepsilon_{2C}^{\text{AS}}| \rangle$  since 76% of the events satisfy this condition.

## V. DISCUSSION AND CONCLUSIONS

In the present study, we estimated the energy cascade rate from both incompressible and compressible cases, using two different closures for the compressible model, i.e., the isothermal and the polytropic case. In particular, we computed the incompressible and compressible energy cascade rate in the solar wind at different heliocentric distances ( $\sim 0.1 - 0.8$  au). First, we found a clear correlation between the energy cascade rate and the distance to the Sun. Our observational results show that, as we get closer to the Sun, the absolute value of the energy cascade rate increase. This observational result is compatible with recent studies (Adhikari *et al.*, 2015; Bandyopad-

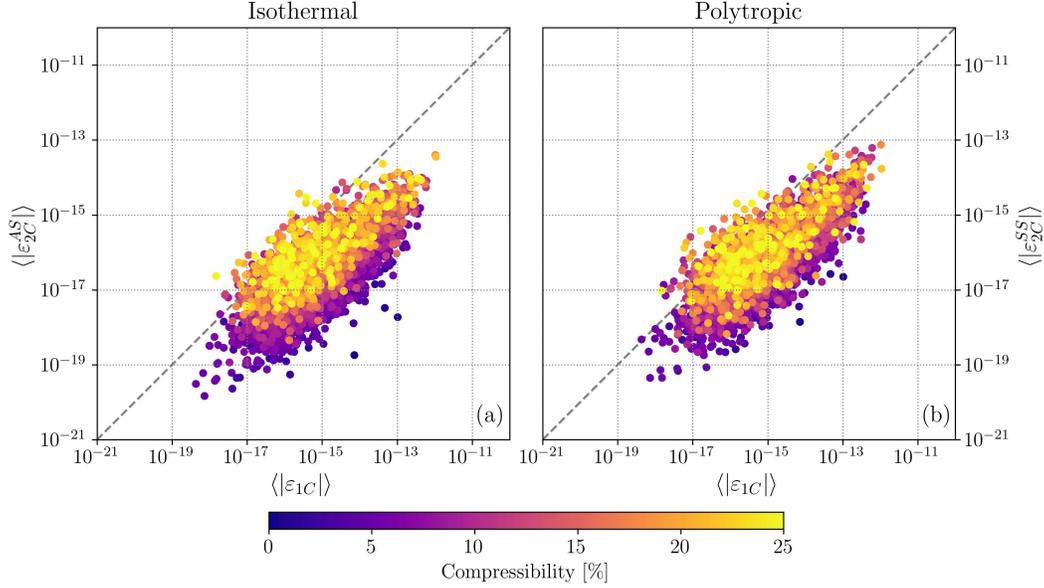


Figure 6. The compressible cascade rate component  $\langle |\varepsilon_{2C}| \rangle$  as a function of the Yaglom-like term  $\langle |\varepsilon_{1C}| \rangle$  in the MHD scales in the case of using an isothermal (AS) and polytropic (SS) model, respectively. The colobar corresponds to the level of compressibility per event.

hyay *et al.*, 2020; Adhikari *et al.*, 2021; Andrés *et al.*, 2021; Andrés *et al.*, 2022). For instance, Andrés *et al.* (2022) showed a correlation between the incompressible energy cascade and the heliocentric distance in the inner heliosphere using isotropic and anisotropic exact relations. Also, Bandyopadhyay *et al.* (2020) have estimated the energy transfer rate from the first PSP perihelion using an incompressible exact relation and the von-Kármán decay law. The authors found that the energy cascade obtained near the perihelion is about 100 times higher than the average value at 1 au. Although we used a more complex theoretical model, as we include density fluctuations to model the expression for the energy cascade, our results show a similar increment of the energy (up to two order of magnitude) as we approach to the Sun, probably due to the increase of the mean value of the plasma density, magnetic and velocity magnitudes.

Secondly, we observed an increment of the compressible energy cascade rate when the level of compressibility increases in the solar wind. We analyzed the competition between the first and second term of the total expression of the compressible cascade for both isothermal and polytropic models. Despite the fact that the first term is dominant in most of the cases, we found that the second term (purely compressible) still plays a relevant role for a proper estimation of the total compressible cascade. Moreover, we obtained that, for the most compressible events (20%-25%),

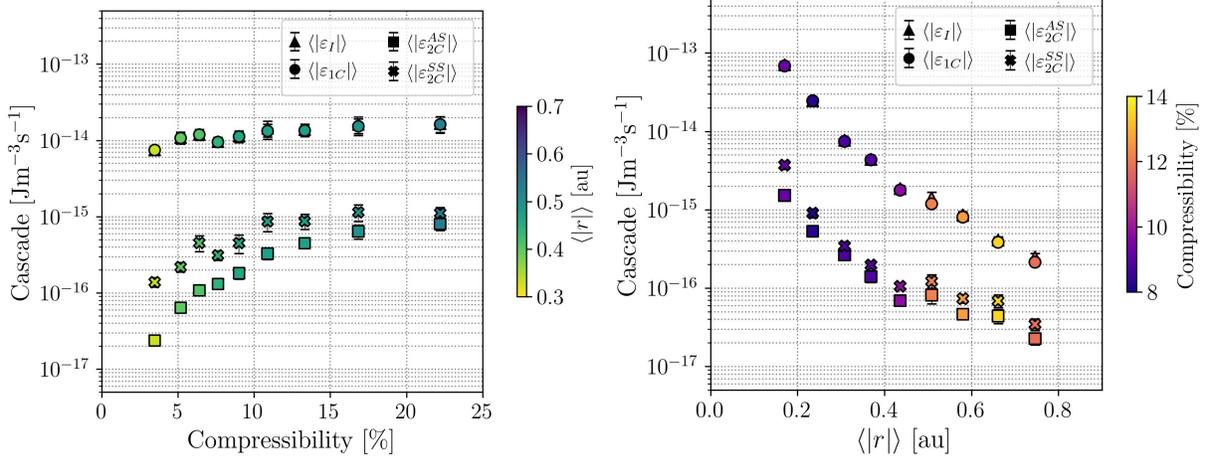


Figure 7. Average of the energy cascade rate for a given: compressibility bin using a colorbar that corresponds to the heliocentric distance (left) and a heliocentric distance bin where the colorbar corresponds to the level of compressibility (right). The total compressible cascade, the compressible component, the Yaglom-like term and the incompressible cascade are included.

there is a non negligible growth in the compressible cascade with respect to the incompressible one. Therefore, higher density fluctuations in the plasma lead to increasing  $\varepsilon_C$  over  $\varepsilon_I$  (Sorriso-Valvo *et al.*, 2007; Marino *et al.*, 2008; Andrés *et al.*, 2019, 2021). Similarly, using THEMIS, MAVEN and PSP observations (at the first encounter), Andrés *et al.* (2021) reported moderate increases of the isothermal compressible cascade with respect to the incompressible one at different heliocentric distances. In our case of study, we expand these previous results including the polytropic model and much more extended data set. Our observational results showed more significant increments between the compressible and incompressible cascade, especially for the less energetic events. In addition, Hadid, Sahraoui, and Galtier (2017) showed that the energy cascade rate increases as compressibility increases in the plasma in the slow solar wind. Note that in the present work, we do not separate the events into fast and slow solar wind but we confirmed this observational results.

Moreover, we reported the average of the energy cascade rate in order to study and compare the behavior between the compressibility levels and the heliocentric distance. We related these two magnitudes and found out that the compressibility increases as we increase the distance to the Sun. Our results are in good agreement with Chen *et al.* (2020), who computed the magnetic compressibility coefficient  $C_B = (\delta|B|/|B|)^2$  and showed a clear decrease toward smaller helio-

centric distances. They observed that the compressibility levels at PSP perihelion are an order of magnitude smaller at 1 au. However, their results are based on magnetic compressibility in the frequency domain unlike the present work where we computed the mean values in the real space. On the other hand, [Adhikari et al. \(2020\)](#) studied the frequency distribution of the solar wind compressibility between 0.17 au and 0.61 au and showed that density fluctuation is concentrated mainly around 0.15 au and decreases with the heliocentric distance. This result is compatible with the density fluctuation levels found by [Andrés et al. \(2021\)](#). Instead, we showed that the density fluctuation reach its highest values approximately at 0.5 au and decreases as we get closer to the Sun.

Finally, we compared the compressible energy cascade calculated by using an isothermal and polytropic model based on previous theoretical works ([Andrés and Sahraoui \(2017\)](#); [Simon and Sahraoui \(2021\)](#)). From the figures mentioned above and, specially, Fig 7, we noticed that there is a clear trend of the polytropic cascade to be larger than the isothermal one. In particular, [Simon and Sahraoui \(2021\)](#) observed the same behavior at MHD scales despite using few events unlike our present statistical work. However, it is worth mentioning that the two compressible models (isothermal and polytropic) give essentially the same cascade rate due to the fact that the contribution of the Yaglom-like term  $\varepsilon_{1C}$  tends to be dominate over  $\varepsilon_{2C}$  in the inertial MHD range in both cases. In summary, our observational results support the idea that the compressible energy cascade rate increases at small heliocentric distance and at large values of compressibility or density fluctuation levels. Nevertheless, there are some aspects of this work that require improvement. First, we found some discrepancies about the intrinsic relation between the heliocentric distances and the compressibility levels. In general, at smaller heliocentric distances, the compressibility and thus, density fluctuations tends to be larger. Therefore, some other process may be acting to suppress these fluctuation as we get closer to the Sun. Second, we only focus in the MHD scales. Recent exact relations could be used to estimate the transfer of energy as we reach the sub ion scales ([Andrés, Galtier, and Sahraoui, 2018](#); [Simon and Sahraoui, 2021, 2022](#)). In forthcoming works, further statistical investigation of these topics are going to be carried out using in situ data in more compressible environments (like the Earth's magnetosheath).

## ACKNOWLEDGMENTS

N.A. and P.D. acknowledge financial support from CNRS/CONICET Laboratoire International Associé (LIA) MAGNETO. N.A. acknowledges financial support from the following grants: PICT 2018-1095 and UBACyT 20020190200035BA. P.D. acknowledges financial support from PIP Grant No. 11220150100324 and 11220200101752 and PICT Grant No. 2018-4298. We thank the NASA Parker Solar Probe SWEAP team led by J. Kasper and FIELDS team led by S. D. Bale for use of data. PSP data are publicly available (<https://research.ssl.berkeley.edu/data/psp/>)

## REFERENCES

- Adhikari, L., Zank, G., Bruno, R., Telloni, D., Hunana, P., Dosch, A., Marino, R., and Hu, Q., “The transport of low-frequency turbulence in astrophysical flows. ii. solutions for the super-alfvénic solar wind,” *The Astrophysical Journal* **805:63**, 18pp (2015).
- Adhikari, L., Zank, G., Zhao, L., Telloni, D., Horbury, T., O’Brien, H., Evans, V., Angelini, V., Owen, C., Louarn, P., and Fedorov, A., “Evolution of anisotropic turbulence in the fast and slow solar wind: Theory and solar orbiter measurements,” *Astronomy and Astrophysics* (2021).
- Adhikari, L., Zank, G. P., Zhao, L.-L., Kasper, J. C., Korreck, K. E., Stevens, M., Case, A. W., Whittlesey, P., Larson, D., Livi, R., *et al.*, “Turbulence transport modeling and first orbit parker solar probe (psp) observations,” *The Astrophysical Journal Supplement Series* **246**, 38 (2020).
- Alexandrova, O., Chen, C. H. K., Sorriso-Valvo, L., Horbury, T. S., and Bale, S. D., “Solar wind turbulence and the role of ion instabilities,” *Space Science Reviews* **178**, 101–139 (2013).
- Andrés, N. and Banerjee, S., “Statistics of incompressible hydrodynamic turbulence: An alternative approach,” *Phys. Rev. Fluids* **4**, 024603 (2019).
- Andrés, N., Galtier, S., and Sahraoui, F., “Exact law for homogeneous compressible hall magnetohydrodynamics turbulence,” *Physical Review E* **97**, 013204 (2018).
- Andrés, N., Romanelli, N., Hadid, L. Z., Sahraoui, F., DiBraccio, G., and Halekas, J., “Solar wind turbulence around mars: Relation between the energy cascade rate and the proton cyclotron waves activity,” *The Astrophysical Journal* **902**, 134 (2020).
- Andrés, N. and Sahraoui, F., “Alternative derivation of exact law for compressible and isothermal magnetohydrodynamics turbulence,” *Physical Review E* **96**, 053205 (2017).
- Andrés, N., Sahraoui, F., Galtier, S., Hadid, L. Z., Ferrand, R., and Huang, S. Y., “Energy cascade

- rate measured in a collisionless space plasma with mms data and compressible hall magnetohydrodynamic turbulence theory,” *Phys. Rev. Lett.* **123**, 245101 (2019).
- Andrés, N., Sahraoui, F., Hadid, L. Z., Huang, S. Y., Romanelli, N., Galtier, S., DiBraccio, G., and Halekas, J., “The evolution of compressible solar wind turbulence in the inner heliosphere: Psp,themis and maven observations,” In press *ApJ* (2021).
- Andrés, N., Sahraoui, F., Huang, S., Hadid, L., and Galtier, S., “The incompressible energy cascade rate in anisotropic solar wind turbulence,” *Astronomy & Astrophysics* **661**, A116 (2022).
- Auster, H. U., Glassmeier, K. H., Magnes, W., Aydogar, O., Baumjohann, W., Constantinescu, D., Fischer, D., Fornacon, K. H., Georgescu, E., Harvey, P., *et al.*, “The themis fluxgate magnetometer,” in *The THEMIS Mission* (Springer, 2009) pp. 235–264.
- Bale, S., Badman, S., Bonnell, J., Bowen, T., Burgess, D., Case, A., Cattell, C., Chandran, B., Chaston, C., Chen, C., *et al.*, “Highly structured slow solar wind emerging from an equatorial coronal hole,” *Nature* **576**, 237–242 (2019).
- Bale, S., Goetz, K., Harvey, P., Turin, P., Bonnell, J., De Wit, T. D., Ergun, R., MacDowall, R., Pulupa, M., André, M., *et al.*, “The fields instrument suite for solar probe plus,” *Space science reviews* **204**, 49–82 (2016).
- Bandyopadhyay, R., Chasapis, A., Chhiber, R., Parashar, T. N., Maruca, B. A., Matthaeus, W. H., Schwartz, S. J., Eriksson, S., Contel, O. L., Breuillard, H., Burch, J. L., Moore, T. E., Pollock, C. J., Giles, B. L., Paterson, W. R., Dorelli, J., Gershman, D. J., Torbert, R. B., Russell, C. T., and Strangeway, R. J., “Solar wind turbulence studies using MMS fast plasma investigation data,” *The Astrophysical Journal* **866**, 81 (2018).
- Bandyopadhyay, R., Goldstein, M., Maruca, B., Matthaeus, W., Parashar, T., Ruffolo, D., Chhiber, R., Usmanov, A., Chasapis, A., Qudsi, R., *et al.*, “Enhanced energy transfer rate in solar wind turbulence observed near the sun from parker solar probe,” *The Astrophysical Journal Supplement Series* **246**, 48 (2020).
- Banerjee, S. and Andrés, N., “Scale-to-scale energy transfer rate in compressible two-fluid plasma turbulence,” *Physical Review E* **101**, 043212 (2020).
- Banerjee, S. and Galtier, S., “Exact relation with two-point correlation functions and phenomenological approach for compressible magnetohydrodynamic turbulence,” *Physical Review E* **87**, 013019 (2013).
- Banerjee, S. and Galtier, S., “A kolmogorov-like exact relation for compressible polytropic turbulence,” *Journal of Fluid Mechanics* **742**, 230–242 (2014).

- Banerjee, S., Hadid, L. Z., Sahraoui, F., and Galtier, S., “Scaling of compressible magnetohydrodynamic turbulence in the fast solar wind,” *The Astrophysical Journal Letters* **829**, L27 (2016).
- Banerjee, S. and Kritsuk, A. G., “Energy transfer in compressible magnetohydrodynamic turbulence for isothermal self-gravitating fluids,” *Phys. Rev. E* **97**, 023107 (2018).
- Boldyrev, S., Mason, J., and Cattaneo, F., “Dynamic alignment and exact scaling laws in magnetohydrodynamic turbulence,” *Astrophys. J. Lett.* **699**, L39 (2009).
- Bruno, R. and Carbone, V., “The solar wind as a turbulence laboratory,” *Living Reviews in Solar Physics* **10**, 1–208 (2013).
- Carbone, V., Marino, R., Sorriso-Valvo, L., Noullez, A., and Bruno, R., “Scaling laws of turbulence and heating of fast solar wind: the role of density fluctuations,” *Physical review letters* **103**, 061102 (2009).
- Case, A. W., Kasper, J. C., Stevens, M. L., Korreck, K. E., Paulson, K., Daigneau, P., Caldwell, D., Freeman, M., Henry, T., Klingensmith, B., *et al.*, “The solar probe cup on the parker solar probe,” *The Astrophysical Journal Supplement Series* **246**, 43 (2020).
- Chen, C., Bale, S., Bonnell, J., Borovikov, D., Bowen, T., Burgess, D., Case, A., Chandran, B., de Wit, T. D., Goetz, K., *et al.*, “The evolution and role of solar wind turbulence in the inner heliosphere,” *The Astrophysical Journal Supplement Series* **246**, 53 (2020).
- Chen, C. H. K., “Recent progress in astrophysical plasma turbulence from solar wind observations,” *Journal of Plasma Physics* **82**, 535820602 (2016).
- Coburn, J. T., Forman, M. A., Smith, C. W., Vasquez, B. J., and Stawarz, J. E., “Third-moment descriptions of the interplanetary turbulent cascade, intermittency and back transfer,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **373**, 20140150 (2015).
- Davies, P. and Gather, U., “The identification of multiple outliers,” *Journal of the American Statistical Association* **88**, 782–792 (1993).
- Ferrand, R., Galtier, S., Sahraoui, F., and Federrath, C., “Compressible turbulence in the interstellar medium: New insights from a high-resolution supersonic turbulence simulation,” *The Astrophysical Journal* **904**, 160 (2020).
- Fox, N., Velli, M., Bale, S., Decker, R., Driesman, A., Howard, R., Kasper, J. C., Kinnison, J., Kusterer, M., Lario, D., *et al.*, “The solar probe plus mission: humanity’s first visit to our star,” *Space Science Reviews* **204**, 7–48 (2016).
- Fraternale, F., Pogorelov, N. V., Richardson, J. D., and Tordella, D., “Magnetic turbulence spectra

- and intermittency in the heliosheath and in the local interstellar medium,” [The Astrophysical Journal](#) **872**, 40 (2019).
- Frisch, U., *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press., 1995).
- Galtier, S. and Banerjee, S., “Exact relation for correlation functions in compressible isothermal turbulence,” *Physical review letters* **107**, 134501 (2011).
- Hadid, L., Sahraoui, F., and Galtier, S., “Energy cascade rate in compressible fast and slow solar wind turbulence,” *The Astrophysical Journal* **838**, 9 (2017).
- Hadid, L., Sahraoui, F., Galtier, S., and Huang, S., “Compressible magnetohydrodynamic turbulence in the earth’s magnetosheath: estimation of the energy cascade rate using in situ spacecraft data,” *Phys. Rev. Lett.* **120**, 055102 (2018).
- Howes, G. G., Bale, S. D., Klein, K. G., Chen, C. H. K., Salem, C. S., and TenBarge, J. M., “The slow-mode nature of compressible wave power in solar wind turbulence,” [The Astrophysical Journal](#) **753**, L19 (2012).
- Huang, S. and Sahraoui, F., “Testing of the taylor frozen-in-flow hypothesis at electron scales in the solar wind turbulence,” *The Astrophysical Journal* **876**, 138 (2019).
- Huang, S., Zhang, J., Sahraoui, F., He, J., Yuan, Z., Andrés, N., Hadid, L., Deng, X., Jiang, K., Yu, L., *et al.*, “Kinetic scale slow solar wind turbulence in the inner heliosphere: Coexistence of kinetic alfvén waves and alfvén ion cyclotron waves,” *The Astrophysical Journal Letters* **897**, L3 (2020).
- von Kármán, T. and Howarth, L., “On the statistical theory of isotropic turbulence,” [Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences](#) **164**, 192–215 (1938).
- Kasper, J. C., Abiad, R., Austin, G., Balat-Pichelin, M., Bale, S. D., Belcher, J. W., Berg, P., Bergner, H., Berthomier, M., Bookbinder, J., *et al.*, “Solar wind electrons alphas and protons (sweap) investigation: design of the solar wind and coronal plasma instrument suite for solar probe plus,” *Space Science Reviews* **204**, 131–186 (2016).
- Kasper, J. C., Bale, S. D., Belcher, J. W., Berthomier, M., Case, A. W., Chandran, B. D., Curtis, D., Gallagher, D., Gary, S., Golub, L., *et al.*, “Alfvénic velocity spikes and rotational flows in the near-sun solar wind,” *Nature* **576**, 228–231 (2019).
- Kiyani, K. H., Osman, K. T., and Chapman, S. C., “Dissipation and heating in solar wind turbulence: from the macro to the micro and back again,” (2015).
- Kritsuk, A. G., Wagner, R., and Norman, M. L., “Energy cascade and scaling in supersonic

- isothermal turbulence,” *Journal of Fluid Mechanics* **729**, R1 (2013).
- Liu, H., Shah, S., and Jiang, W., “On-line outlier detection and data cleaning,” *Computers and Chemical Engineering* **28**, 1635–1647 (2004).
- MacBride, B. T., Smith, C. W., and Forman, M. A., *Astrophys. J.* **679**, 1644–1660 (2008).
- Marino, R., Sorriso-Valvo, L., Carbone, V., Noullez, A., Bruno, R., and Bavassano, B., “Heating the Solar Wind by a Magnetohydrodynamic Turbulent Energy Cascade,” *Astrophys. J. Lett.* **677**, L71–L74 (2008).
- Marsch, E. and Mangeney, A., “Ideal mhd equations in terms of compressive Elsässer variables,” *Journal of Geophysical Research: Space Physics* **92**, 7363–7367 (1987).
- Marsch, E., Mühlhäuser, K.-H., Schwenn, R., Rosenbauer, H., Pilipp, W., and Neubauer, F. M., “Solar wind protons: Three-dimensional velocity distributions and derived plasma parameters measured between 0.3 and 1 au,” *Journal of Geophysical Research: Space Physics* **87**, 52–72 (1982), <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/JA087iA01p00052>.
- Masters, A., Achilleos, N., Dougherty, M. K., Slavin, J. A., Hospodarsky, G. B., Arridge, C. S., and Coates, A. J., “An empirical model of Saturn’s bow shock: Cassini observations of shock location and shape,” *Journal of Geophysical Research (Space Physics)* **113**, A10210 (2008).
- Matthaeus, W. and Velli, M., “Who needs turbulence?” *Space science reviews* **160**, 145 (2011).
- Matthaeus, W. H., Zank, G. P., Smith, C. W., and Oughton, S., *Phys. Rev. Lett.* **82**, 3444– (1999).
- Mininni, P. D. and Pouquet, A., “Finite dissipation and intermittency in magnetohydrodynamics,” *Phys. Rev. E* **80**, 025401 (2009).
- Monin, A. S. and Yaglom, A. M., *Statistical Fluid Mechanics: Mechanics of Turbulence*, Vol. 2 (Cambridge, MA: MIT Press., 1975).
- Osman, K. T., Wan, M., Matthaeus, W. H., Weygand, J. M., and Dasso, S., “Anisotropic third-moment estimates of the energy cascade in solar wind turbulence using multispacecraft data,” *Phys. Rev. Lett.* **107**, 165001 (2011).
- Parashar, T., Goldstein, M., Maruca, B., Matthaeus, W., Ruffolo, D., Bandyopadhyay, R., Chhiber, R., Chasapis, A., Qudsi, R., Vech, D., *et al.*, “Measures of scale-dependent alfvénicity in the first PSP solar encounter,” *The Astrophysical Journal Supplement Series* **246**, 58 (2020).
- Pearson, R., Neuvo, Y., Astola, J., and Gabbouj, M., “Generalized hamper filters,” *EURASIP Journal on Advances in Signal Processing* **2016** (2016), [10.1186/s13634-016-0383-6](https://doi.org/10.1186/s13634-016-0383-6).
- Pine, Z. B., Smith, C. W., Hollick, S. J., Argall, M. R., Vasquez, B. J., Isenberg, P. A., Schwadron, N. A., Joyce, C. J., Sokół, J. M., Bzowski, M., Kubiak, M. A., Hamilton, K. E., McLaurin,

- M. L., and Leamon, R. J., “Solar wind turbulence from 1 to 45 au. i. evidence for dissipation of magnetic fluctuations using voyager and ACE observations,” [The Astrophysical Journal](#) **900**, 91 (2020).
- Politano, H. and Pouquet, A., “Dynamical length scales for turbulent magnetized flows,” *Geophysical Research Letters* **25**, 273–276 (1998a).
- Politano, H. and Pouquet, A., “von kármán–howarth equation for magnetohydrodynamics and its consequences on third-order longitudinal structure and correlation functions,” *Physical Review E* **57**, R21 (1998b).
- Sahraoui, F., “Diagnosis of magnetic structures and intermittency in space-plasma turbulence using the technique of surrogate data,” *Phys. Rev. E* **78**, 026402 (2008).
- Sahraoui, F., Hadid, L., and Huang, S., “Magnetohydrodynamic and kinetic scale turbulence in the near-earth space plasmas: a (short) biased review,” *Reviews of Modern Plasma Physics* **4**, 1–33 (2020).
- Simakov, A. N. and Chacón, L., “Quantitative, comprehensive, analytical model for magnetic reconnection in hall magnetohydrodynamics,” [Phys. Rev. Lett.](#) **101**, 105003 (2008).
- Simon, P. and Sahraoui, F., “General exact law of compressible isentropic magnetohydrodynamic flows: theory and spacecraft observations in the solar wind,” *The Astrophysical Journal* **916**, 49 (2021).
- Simon, P. and Sahraoui, F., “Exact law for compressible pressure-anisotropic magnetohydrodynamic turbulence: Toward linking energy cascade and instabilities,” [Phys. Rev. E](#) **105**, 055111 (2022).
- Sorriso-Valvo, L., Marino, R., Carbone, V., Noullez, A., Lepreti, F., Veltri, P., Bruno, R., Bavasano, B., and Pietropaolo, E., “Observation of inertial energy cascade in interplanetary space plasma,” *Physical review letters* **99**, 115001 (2007).
- Stawarz, J. E., Smith, C. W., Vasquez, B. J., Forman, M. A., and MacBride, B. T., “The turbulent cascade and proton heating in the solar wind at 1 au,” *The Astrophysical Journal* **697**, 1119 (2009).
- Stawarz, J. E., Smith, C. W., Vasquez, B. J., Forman, M. A., and MacBride, B. T., “The turbulent cascade for high cross-helicity states at 1 au,” *The Astrophysical Journal* **713**, 920 (2010).
- Tu, C.-Y. and Marsch, E., “Mhd structures, waves and turbulence in the solar wind: Observations and theories,” [Space Science Reviews](#) **73**, 1–210 (1995).
- Vasquez, B. J., Smith, C. W., Hamilton, K., MacBride, B. T., and Leamon, R. J., “Evalua-

tion of the turbulent energy cascade rates from the upper inertial range in the solar wind at 1 au,” *Journal of Geophysical Research: Space Physics* **112** (2007), 10.1029/2007JA012305, <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2007JA012305>.

Wan, M., Servidio, S., Oughton, S., and Matthaeus, W. H., “The third-order law for magnetohydrodynamic turbulence with shear: Numerical investigation,” *Phys. Plasmas* **17**, 052307 (2010).

Weygand, J. M., Matthaeus, W. H., Dasso, S., Kivelson, M. G., and Walker, R. J., “Taylor scale and effective magnetic Reynolds number determination from plasma sheet and solar wind magnetic field fluctuations,” *J. Geophys. Res.: Space Phys.* **112**, A10 (2007).